

**UNIVERSITY TUNKU ABDUL RAHMAN**

**FACULTY OF SCIENCE**

**UDPS 2293 QUEUING MODELS**

**ASSIGNMENT**

**TRIMESTER JANUARY 2019**

**LOCATION: AUTOMATIC CAR WASHING SYSTEM**

**LECTURER: MR LIEW KAH FEI**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
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**1.0 Introduction**

Queuing model is a model that defined as a mathematical problem that deals with the problem of queues or waiting lines. This allow one to predict the length of the queues so that they can plan efficiently and execute business decisions appropriately.

In our assignment, we are going to observe the queuing line of a system and collect the data in a reasonable time period. We target our queuing system on single server model which is serving in a single line with only one server. Therefore, the system we choose is automatic car washing system, which the shop named as “Drive Thru Car Wash”. The location of the system is placed at Taman Melayu Jaya, 31900 Kampar, Perak. It is a single server model which allows customer to drive thru the machine to wash their car.

In this system, customer will reach in front of the machine and wait for the instruction from the workers before going into the machine. Thus, the Kendall notation for the system is M/M/1/GD/∞/∞. M implies the exponential distribution. The M/M/1 notation implies that it is an exponentially distributed inter-arrival times, exponentially distributed service times and it is a single server queue. Whereas for GD which is refers to general discipline, means first come first serve service discipline. GD/∞/∞ implies that a general discipline system there is no limit on the maximum number in the system (infinity) and infinite population of potential customers. In this study, we will find the expected number of customers in system (), expected number of customers in queue (), expected waiting time in system () and expected waiting time in queue (). After the data collection, we also will determine the arrival rate () and the service rate () and do the simulation to estimate the value of.

The objectives of this study is to investigate and analyze the queuing system of automatic car washing machine in term of its arrival rate, service rate, the characteristics and performance of the queue system and the percentage of time that the server is idle. We also compare the queuing system between weekday and weekend on the evening which is from 4.00 p.m. to 6.00 p.m. Therefore, through this study we will determine the least queuing day for customer to go for a car wash.

**2.0 Data and Methodology**

In this research, we are going to analyze and determine the characteristics and performances of the queuing system of the automatic car washing machine. This model was implied the M/M/1/GD// Kendall-lee notation to perform this queuing system which is first come first serve discipline and there is no limit on the maximum number in the system and it is infinite population of potential customers. In this model, the = as there is no found ( due to infinite number of customers so no customer lost.

There are some assumptions when we study in the research. Firstly, the source population has infinite size. Moreover, the arrivals queue in single waiting line and single service line and are served on a first-in, first-out (FIFO) basis, every arrival waits to be served regardless of the length of the line or queue. Arrivals are independent of preceding arrivals, but the average number of arrivals (arrival rate) does not change over time. The customers who leaving in the queue do not involve in the data calculation. In this system, the drying and vacuum services are not involved in the service time.

To collect the data for this study, we choose to take two days for observations which are weekday and weekend. We start to collect the data on 15/03/2019 (Friday) as weekday and 16/03/2019 (Saturday) as weekend. The data are collected around 2 hours each day which is between 4.00 p.m. to 6.00 p.m. When the car drive into the lane that before going to the machine, we known it as arrival time. When the car start to go into the machine, we record the time as the service time until finish all the washing service and cleaning service. Leaving time is recorded when the car come out from the machine. After the observation and all the data have been collected, we calculated the inter-arrival time by calculating the difference of the arrival time of the current customer and the previous customer. We also calculated the service time by calculating the difference of the leaving time and the service time that that we recorded. Hence, we compute the arrival rate, and the service rate, by using the formula below:

*\*\*The unit for and is unit per minute. We also have convert the unit to per hour by multiply them with 60.*

After done for the calculation of arrival rate and service rate, we perform the simulation technique to compute the steady-state measures of performance by using “Microsoft Excel”. In order to achieve steady-state measures, we obtain 2000 samples with all the arrival process, waiting process and serving process which consists of arrival time, inter-arrival time, service time, and waiting time. In order to perform the simulation of this study, we have to calculate the random inter-arrival time and service time by using these formulas below:

*\*\*The and are obtained from the observations of our study and rand( ) is the formula to generate random number in “Microsoft Excel”.*

After the inter-arrival time and service time are generated, we can obtain the ordering time, leaving time, waiting time in the system and waiting time in the queue. Then, we calculate the simulated average waiting time in the queue to compare the difference of performance between the model that derived based on observations and the simulated model.

**3.0 Result and Analysis**

Figure 1 in appendix shows the histogram of service time in seconds and the frequency of customers on Friday whereas Figure 2 in appendix shows the histogram of inter-arrival time in seconds and the frequency of customer on Friday. From the histograms, the service times and inter-arrival times are rarely constant. In this case, the number of arrivals in a given time interval has Poisson distribution. Moreover, inter-arrival times can be shown to have the exponential distribution. The figures above proven that the inter-arrival times have an exponential distribution. This can also be explained as the inter-arrival times are independent random variables, as they follow the exponential distribution.

Figure 3 in appendix shows the histogram of the service time in seconds and frequency of the customers on Saturday while Figure 4 in appendix shows the histogram of the inter-arrival time in seconds and frequency of the customers in a fixed time period on Saturday. Both histogram graphs above have shown that the service time and inter-arrival time follow the exponential distribution. Therefore, it fulfils the assumptions.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | (per minute) | (per minute) |  |  |
| Friday | 0.411 | 0.7317 | 0.5616 | 0.4384 |
| Saturday | 0.6122 | 0.7229 | 0.8469 | 0.1531 |

Table 1.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | *C* |
| Friday | 1.281 | 0.7196 | 3.1177 | 1.751 | 0.562 |
| Saturday | 5.533 | 4.6864 | 9.0378 | 7.6544 | 0.847 |

Table 2.

Based on the table above, we found that the average number of customers arriving per minute, on weekday while the mean arrival rate of customers on weekend is . On average the number of customers that can be served per minute on weekday is while on weekend is . Although there is not a huge difference in the mean service rate for both weekday and weekend, but we can see that the service rate on weekday is slightly longer than weekend. The probability of no car in the system () is much higher on Friday (0.4384) compare to Saturday (0.1531).

As a result, the average number of customers in the service system, , the average number of customers waiting in line, , the average time spent waiting in the system, , the average time spent waiting in line, , on Saturday are all higher compare to Friday. This indicates that there are more customers in the system and queue as well as longer time spending in the system and waiting in queue on weekend. In other words, the expected number of busy servers, *C*, on weekday is 0.562 while the expected number of busy servers on weekend is 0.847. This shows that the chances for the server to be busy is higher on weekend compare to weekday.

The percentage of time that the server will idle on Friday (Weekday) is 43.84% while the percentage of time that the server will idle on Saturday (Weekend) is 15.31%. This can be further explained as 43.81% of time the automatic car washing machine doesn’t has customer on weekday, however, there is 15.31% of time that the system is idle on weekend. The result indicates that a higher chance to have the server is empty on weekday compare to weekend.

Besides, we use Microsoft Excel to simulate the data for the automatic car washing machine. We simulated the data with 500 random numbers for the inter-arrival time and service time for each day (Friday and Saturday) respectively. We would able to verify the outcome of the analytical model that we have obtained by this simulation model. The simulation model will be able to reflect the actual operation of the automatic car washing machine more closely for the whole day. Simulation model is considered to be a long run system. It usually closer to the actual system and will increase the accuracy. This is because our data may be not accurate because we only collect the data for two hours on Friday and Saturday respectively.

From the simulated data, we obtained the graph as shown in appendix Figure 5, Figure 6, Figure 7, and Figure 8 for the average of waiting time in system () and the average of waiting time in queue ().

From the simulated data for Friday, we can obtained the average waiting time in system, is equal to 3.0167 minutes. However, from the observation, we obtained the value of is equal to 3.1177 minutes. The error percentage we obtained from is equal to 3.24% which means that this stimulation is acceptable. We found that there are 0.8% of the customers have zero waiting time. This means that 4 out of 500 customers are no need to spend their time to wait in the system.

Besides from the average waiting time in system, we also obtained the average waiting time in queue, is equal to 1.6233 minutes. But the value of that we have obtained from the observation is equal to 1.7540 minutes. The error percentage we obtained from is equal to 3.87%. We also found that there are 41.2% of the customers are being served immediately when they arrived to the automatic car washing machine.

From the simulated data from Saturday, we can obtained the average waiting time in system, is equal to 12.3667 minutes. However, from the observation, we obtained the value of is equal to 9.0378 minutes. The error percentage we obtained from is equal to 36.83% which means that this stimulation is not acceptable. We found that there are none of the customers have zero waiting time. It means that all customers have to wait to be served in the system.

Besides from the average waiting time in system, we also obtained the average waiting time in queue, is equal to 10.8833 minutes. But that we have obtained from the observation is equal to 7.6544 minutes. The error percentage we obtained from is equal to 42.18%. We also found that there are 7.2% of the customers are served immediately without spending their time in the queue.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Performance measure | Friday | | | Saturday | | |
| Collected data | Simulated data | Error percentage (%) | Collected data | Simulated data | Error percentage (%) |
| Average() | 3.1177 | 3.0167 | 3.24 | 9.0378 | 12.3667 | 36.83 |
| Average() | 1.754 | 1.6233 | 3.87 | 7.6544 | 10.8833 | 42.18 |

**Table 3:** The comparison for the average and for the collected data and simulated data on Friday and Saturday

**4.0 Conclusion and Recommendation**

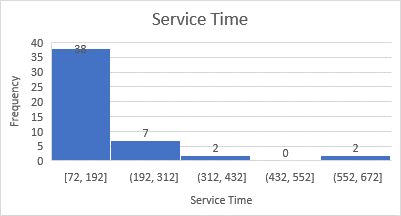
In conclusion, on weekday, is 3.118 minutes and is 1.751 minutes. In words, the customer is expected to spend around 3.118 minutes in the automatic car washing system and have to wait around 1.751 minutes in the queue before entering the system. In addition, and are 1.281 and 0.72 respectively. On average, there is 1.281 customers in the system and 0.72 customer in the queuing line. The percentage of time that the system is idle is 43.84%.

Moreover, on weekend, and observed are 9.038 minutes and 7.654 minutes respectively. This represents that on average, the customer has to spend 9.038 minutes in the system and 7.654 minutes in the queuing line. Besides, is 5.533 and is 4.686 which means 5.533 customers are expected to be in the system and 4.686 customers in the queue. The percentage of time that the system is idle is 15.31%.

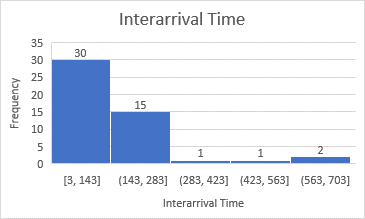
For the simulation data, from Figure 5 and Figure 6 in appendix, steady state is observed and both graphs converge to one value in the end. Numerically, we got the values by calculating it using the formula and obtained that the value of and are approximate to 3.017 minutes and 1.683 minutes respectively on weekday. Whereas on weekend, from Figure 7 and Figure 8, it is observed that the value of is 12.37 minutes and is 10.88 minutes approximately.

From the results for both collected data and simulation data, the automatic car washing system has more customer in the weekend and the customer has to wait longer before their car gets to wash compared to the system on weekday. Thus, we recommend the customer go to the automatic car washing shop on weekday if they want to wash their car. Also, we recommend the shop to open one more automatic car wash system after 5.00 p.m. to reduce the waiting time of the customer in the queue as we observed that more customer come to wash car after 5 p.m. By doing this, they also can increase their service rate.

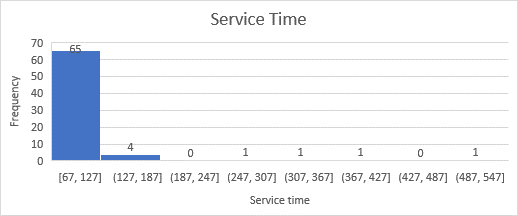
**5.0 Appendix**



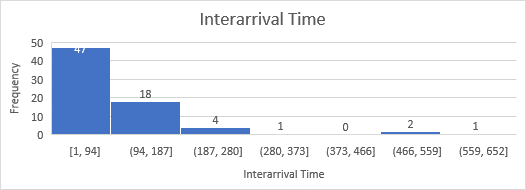
**Figure 1**



**Figure 2**



**Figure 3**



**Figure 4**

For Friday

**Figure 5:** The simulated average for Friday

**Figure 6:** The simulated average for Friday

For Saturday

**Figure 7:** The simulated average for Saturday

**Figure 8:** The simulated average for Saturday

Table 4 shows that the data collected on weekday, which is on 15/03/2019 (Friday) from 4.00 p.m. to 6.00 p.m. We collected 49 data on that day.

|  |  |  |  |
| --- | --- | --- | --- |
| **Customer** | **Arrival Time** | **Service Time** | **Leaving Time** |
| WNQ | 16:01:41 | 16:01:50 | 16:03:15 |
| AFP | 16:03:29 | 16:03:59 | 16:05:25 |
| AKW | 16:13:58 | 16:14:17 | 16:15:47 |
| WHT | 16:16:40 | 16:16:55 | 16:18:21 |
| WJK | 16:18:22 | 16:18:33 | 16:19:57 |
| VF | 16:19:15 | 16:19:57 | 16:21:20 |
| AKP | 16:20:40 | 16:21:20 | 16:22:39 |
| PKA | 16:20:44 | 16:22:39 | 16:24:06 |
| AJN | 16:23:08 | 16:24:06 | 16:25:29 |
| WUK | 16:27:25 | 16:27:45 | 16:29:06 |
| PKX | 16:27:30 | 16:29:06 | 16:30:29 |
| VK | 16:29:16 | 16:30:29 | 16:31:51 |
| WGM | 16:33:35 | 16:33:58 | 16:35:19 |
| PHA | 16:34:33 | 16:35:19 | 16:36:43 |
| VAL | 16:38:08 | 16:38:29 | 16:39:51 |
| PCW | 16:38:38 | 16:39:51 | 16:41:12 |
| AKV | 16:45:08 | 16:45:15 | 16:46:39 |
| AJP | 16:47:05 | 16:47:19 | 16:48:45 |
| TBW | 16:48:10 | 16:48:45 | 16:50:05 |
| WFR | 16:48:32 | 16:50:05 | 16:51:30 |
| AJC | 16:56:49 | 16:57:10 | 16:58:30 |
| CRV | 16:57:50 | 16:58:30 | 16:59:51 |
| AJX | 16:59:58 | 17:00:08 | 17:01:33 |
| DDD | 17:10:09 | 17:10:14 | 17:11:35 |
| AJF | 17:13:38 | 17:13:46 | 17:15:09 |
| WNU | 17:15:31 | 17:15:44 | 17:17:00 |
| AFF | 17:19:47 | 17:20:02 | 17:21:28 |
| WB | 17:24:12 | 17:24:43 | 17:26:07 |
| WLV | 17:24:30 | 17:26:07 | 17:27:28 |
| AKK | 17:27:11 | 17:27:28 | 17:28:51 |
| WUR | 17:30:01 | 17:30:16 | 17:31:40 |
| WTG | 17:31:40 | 17:32:21 | 17:33:42 |
| CCS | 17:35:22 | 17:35:42 | 17:37:02 |
| WPF | 17:35:27 | 17:37:02 | 17:38:26 |
| BNQ | 17:38:01 | 17:38:26 | 17:39:53 |
| PGL | 17:38:10 | 17:39:53 | 17:41:16 |
| AKE | 17:41:56 | 17:42:10 | 17:43:33 |
| AHK | 17:43:56 | 17:44:15 | 17:45:40 |
| ANQ | 17:44:41 | 17:45:40 | 17:47:08 |
| DDG | 17:48:29 | 17:48:58 | 17:50:17 |
| AEQ | 17:50:27 | 17:50:50 | 17:52:17 |
| AHL | 17:50:35 | 17:52:17 | 17:53:40 |
| AJJ | 17:53:10 | 17:53:40 | 17:54:52 |
| AGV | 17:53:42 | 17:54:52 | 17:56:15 |
| AJT | 17:55:25 | 17:56:15 | 17:57:30 |
| AKT | 17:55:29 | 17:57:30 | 17:58:46 |
| AHR | 17:55:32 | 17:58:46 | 17:59:58 |
| AKE | 17:56:54 | 17:59:58 | 18:01:23 |
| AKL | 17:58:54 | 18:01:23 | 18:02:45 |

**Table 4**

Table 5 shows that the data collected on weekend, which is on 16/03/2019 (Saturday) from 4.00 p.m. to 6.00 p.m. We collected 73 data on that day.

|  |  |  |  |
| --- | --- | --- | --- |
| **Customer** | **Arrival Time** | **Service Time** | **Leaving Time** |
| AHN | 16:00:56 | 16:01:07 | 16:02:36 |
| PLV | 16:01:24 | 16:02:36 | 16:03:58 |
| WUL | 16:02:05 | 16:03:58 | 16:05:26 |
| WB | 16:04:06 | 16:05:26 | 16:06:54 |
| PNM | 16:05:21 | 16:06:54 | 16:08:21 |
| WLB | 16:06:13 | 16:08:21 | 16:09:47 |
| AKS | 16:07:16 | 16:09:47 | 16:11:12 |
| AHJ | 16:07:41 | 16:11:12 | 16:12:32 |
| KEH | 16:07:57 | 16:12:32 | 16:13:59 |
| AKE | 16:09:46 | 16:13:59 | 16:15:20 |
| VBR | 16:09:58 | 16:15:20 | 16:16:41 |
| AJX | 16:10:21 | 16:16:41 | 16:17:58 |
| AJL | 16:12:32 | 16:17:58 | 16:19:22 |
| ADA | 16:15:57 | 16:19:22 | 16:20:46 |
| ALE | 16:16:02 | 16:20:46 | 16:22:05 |
| JFK | 16:17:32 | 16:22:05 | 16:23:34 |
| WM | 16:21:22 | 16:23:34 | 16:25:07 |
| AHF | 16:23:09 | 16:25:07 | 16:26:35 |
| BPQ | 16:24:32 | 16:26:35 | 16:28:07 |
| ALB | 16:25:21 | 16:28:07 | 16:29:27 |
| AJB | 16:25:53 | 16:29:27 | 16:30:43 |
| AJT | 16:27:07 | 16:30:43 | 16:32:08 |
| AFT | 16:36:42 | 16:36:53 | 16:38:19 |
| WVB | 16:39:11 | 16:39:30 | 16:40:56 |
| AJG | 16:39:14 | 16:40:56 | 16:42:17 |
| AKB | 16:40:06 | 16:42:17 | 16:43:39 |
| CDN | 16:42:42 | 16:43:39 | 16:44:58 |
| AGJ | 16:43:46 | 16:44:58 | 16:46:14 |
| AKH | 16:45:24 | 16:46:14 | 16:47:41 |
| AFL | 16:48:01 | 16:48:51 | 16:50:13 |
| JPB | 16:52:54 | 16:53:24 | 16:54:43 |
| CDV | 16:55:21 | 16:55:30 | 16:56:56 |
| AHV | 16:55:24 | 16:56:56 | 16:58:24 |
| AKJ | 16:58:01 | 16:58:24 | 16:59:48 |
| WC | 16:58:09 | 16:59:48 | 17:01:11 |
| AKP | 17:02:21 | 17:02:49 | 17:04:16 |
| AKY | 17:05:12 | 17:05:36 | 17:07:03 |
| VAP | 17:13:47 | 17:13:56 | 17:15:24 |
| WGC | 17:14:55 | 17:15:24 | 17:16:46 |
| AJE | 17:15:17 | 17:16:46 | 17:18:06 |
| AHH | 17:15:18 | 17:18:06 | 17:19:30 |
| DAK | 17:23:22 | 17:24:11 | 17:25:45 |
| ALD | 17:25:24 | 17:25:45 | 17:27:02 |
| AFB | 17:26:13 | 17:27:02 | 17:28:21 |
| MBG | 17:27:03 | 17:28:21 | 17:29:46 |
| WHITE | 17:28:26 | 17:29:46 | 17:31:06 |
| WTV | 17:28:51 | 17:31:06 | 17:32:32 |
| JSP | 17:30:23 | 17:32:32 | 17:33:57 |
| KDE | 17:30:26 | 17:33:57 | 17:35:15 |
| WTT | 17:34:24 | 17:35:15 | 17:36:37 |
| AHU | 17:35:06 | 17:36:37 | 17:37:55 |
| WVQ | 17:37:01 | 17:37:55 | 17:39:12 |
| WPM | 17:37:04 | 17:39:12 | 17:40:36 |
| AJM | 17:39:37 | 17:40:36 | 17:41:59 |
| CDF | 17:40:17 | 17:41:59 | 17:43:25 |
| AJL | 17:40:51 | 17:43:25 | 17:44:49 |
| AKE | 17:42:30 | 17:44:49 | 17:46:13 |
| AJA | 17:43:03 | 17:46:13 | 17:47:44 |
| AJV | 17:43:07 | 17:47:44 | 17:49:05 |
| AJH | 17:44:05 | 17:49:05 | 17:50:28 |
| WKV | 17:45:14 | 17:50:28 | 17:51:52 |
| AHM | 17:45:31 | 17:51:52 | 17:53:12 |
| WUW | 17:48:15 | 17:53:12 | 17:54:45 |
| AHF | 17:49:44 | 17:54:45 | 17:56:02 |
| AEH | 17:51:14 | 17:56:02 | 17:57:25 |
| MBP | 17:51:17 | 17:57:25 | 17:58:40 |
| AHQ | 17:51:21 | 17:58:40 | 18:00:08 |
| AJT | 17:52:36 | 18:00:08 | 18:01:27 |
| KBS | 17:55:08 | 18:01:27 | 18:02:48 |
| AED | 17:55:35 | 18:02:48 | 18:04:06 |
| AJJ | 17:56:53 | 18:04:06 | 18:05:34 |
| WC | 17:58:53 | 18:05:34 | 18:06:51 |
| AJM | 17:59:41 | 18:06:51 | 18:08:14 |

**Table 5**